VIVIANA GRASSELLI

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RESEARCH EXPERIENCE

Postdoctoral researcher *2023 - present*
Institut Elie Cartan de Lorraine, Metz, France

Member of the ANR-DFG French German project "Effective Approximation and Dynamics of Many-Body Quantum Systems "

Advisors: Sébastien Breteaux and Jérémy Faupin

Ph.D. program under the supervision of Prof. Jean-Marc Bouclet *2020 - 2023*
Institut de Mathématiques de Toulouse, France

Subject : "Study of the resolvent of the perturbed Schrödinger operator "

EDUCATION AND DEGREES

PhD in Mathematics *2023*
Université Paul Sabatier, Toulouse, France

Master's degree in Mathematics *2020*
Università di Pisa, Italy

Final degree mark: 110/110 (with honours)

Bachelor's degree in Mathematics *2017*
Università degli Studi di Perugia, Italy

Final degree mark: 110/110 (with honours)

TEACHING EXPERIENCE

Université de Lorraine, Metz France *2023 - 2024*

- Mathematical tools course for the Bachelor's degree in Computer Science (*30h*)

Université Paul Sabatier, Toulouse, France *2022 - 2023*

- Tutoring for Mathematics course of Bachelor's degree in Fundamental Sciences (*60h*)
- Numerical practice sessions for Linear Algebra course of Bachelor's degree in Fundamental Sciences (*4h*)

Université Paul Sabatier, Toulouse, France *2021 - 2022*

- Tutoring for Mathematics course of Bachelor's degree in Fundamental Sciences (*30h*)
- Linear algebra course for the Bachelor's degree in Civil and Mechanical Engineering (*30h)*

Università di Pisa, Italy *2019 - 2020*

- Tutoring for Mathematics course of the Bachelor's degree in Economics (*60h)*

RESEARCH INTERESTS

My research lies in the field of mathematical physics and in particular **quantum dynamics**. On one hand I have been studying **spectral properties** for perturbed Hamiltonians of systems in various manifolds settings. This spectral analysis is tightly linked to the dynamical behaviour of the systems. On the other hand, my postdoc focuses on Hamiltonians for systems with **many particles**. My main interest is to obtain properties which are uniform in the number of particles, which will allow us to take the **mean field limit** and deduce properties for non linear models, like the Hartree equation.

PREPRINTS AND PUBLICATIONS

On the definition of zero resonances for the Schrödinger operator with optimal scaling potentials *2023*
To appear in Osaka Journal of Mathematics

Dispersive equations on asymptotically conical manifolds: time decay in the low frequency regime *2023*
Annals of Global Analysis and Geometry

INVITED TALKS

Séminaire de Physique Mathématiques *04/2024*
Institut Camille Jordan, Lyon, France

Journées Jeunes EDPistes en France *03/2024*
Institut de Mathématiques de Toulouse, France

Séminaire Équations aux dérivées partielles *02/2024*
IRMAR, Rennes, France

Séminaire Dynamique Quantique et Classique *01/2024*
Centre de Physique Théorique, Marseille, France

Séminaire EDP et Physique mathématique *01/2024*
LAGA, Université Sorbonne Paris Nord, Paris, France

Kick-off Meeting of the ANR-DFG project Effective Approximation and Dynamics of Many-Body Quantum Systems *08/2023*
Institut Elie Cartan de Lorraine, Metz, France

Séminaire EDP, Analyse et Applications *06/2023*
Institut Elie Cartan de Lorraine, Metz, France

Differential Equation Seminar *05/2023*
Department of Mathematics Graduate School of Science Osaka University, Osaka, Japan

Séminaire Problèmes Spectraux en Physique Mathématique *12/2022*
Institut Henri Poincaré, Paris, France

IMT PDEs students seminar *06/2022*
Institut de Mathématiques de Toulouse, France

IMT Analysis seminar *05/2022*
Institut de Mathématiques de Toulouse, France

ORGANIZING RESPONSABILITIES

Co-organizer of the PhD student seminar *2021-2023*
Institut de Mathématiques de Toulouse

Co-manager of the IMT- EUR MINT- Labex CIMI Twitter account *2021-2023*

PROGRAMMING SKILLS

C, Python, Matlab, Latex

SKILLS

Language	Italian (Native speaker), English (fluent), French (fluent)	
Certifications	IELTS certificate, Overall Band Score: 8, CEFR Level: C1	*2020*

RESEARCH

I. SPECTRAL THEORY

In this part of my research I have focused on spectral properties for the Schrödinger operator, which models the dynamics of a quantum particle. In quantum dynamics, the state of a system is described via a probability distribution which is solution to the Schrödinger equation

$$i\partial_t \psi = P\psi,$$

where we denote by P the Schrödinger operator. This operator describes the energies acting on the system, that is the kinetic and potential energy. In the simplest case, we have $P = -\Delta$ when we look at a particle on $\mathbb{R}^n$ with only kinetic energy.

In my work I have been interested in the Schrödinger operator with several types of perturbations. On one hand I have considered perturbations by metrics, meaning that we look at the problem not on $\mathbb{R}^n$ (a flat space), but on manifolds with curved geometries. On the other hand I have also considered different types of potential perturbations, the electric and the magnetic potential.

The type of manifold under consideration is

$$K \cup (R, \infty) \times S$$

where K is compact region and the unbounded part of the manifold has a product structure $(R, \infty) \times S$, with $R > 0$ and S an $n-1$ dimensional closed manifold. This model includes the basic example of a compact perturbation of $\mathbb{R}^n$, taking for example $K \subset B(0, R)$ the compact perturbation and $(R, \infty) \times S = (R, \infty) \times \mathbb{S}^{n-1}$ the plane $\mathbb{R}^n$ outside of the ball. In my thesis I have considered the cases where $(R, \infty) \times S$ is the perturbation of a cone or of a hyperbolic space? end? .

We consider P a selfadjoint operator, then its resolvent $(P-z)^{-1}$ is well defined in $\mathbb{C} \setminus \mathbb{R}$. The object in my thesis was to study the behaviour of this resolvent when z approaches the real axis and hence the resolvent becomes singular.

For the low frequency regime I have studied the case where P includes the kinetic energy and an electric potential, so P is a Laplace-Beltrami operator on the manifold with a multiplicative potential. In [] I have proved that the limits of the resolvents when $z \to \mathbb{R}^+$, as well as its powers, are bounded operator in suitable weighted spaces. This in turn is applied to derive dynamical properties for the solutions of the Schrödinger equation. Thanks to the relation

$$e^{itP} = \int_0^\infty e^{it\lambda} \lim_{\varepsilon \to 0} ((P - \lambda + i\varepsilon)^{-1} - (P - \lambda - i\varepsilon)^{-1}) d\lambda$$

I have proved decay in time for the L^2 norm of the propagator e^{itP} when localised at low frequencies providing an optimal rate of decay, as well as results for the propagators of the wave and Klein-Gordon equations. These results of local energy decay place themselves in the family of dispersive estimates.

For the high frequency regime I have studied a system where the particle is subject to the kinetic energy, an electric potential energy and a magnetic field. This translates to the fact that P is a Laplace-Beltrami operator with a potential of order zero and a magnetic potential. In this case I have again proved existence for the limits of the resolvent of P, with the addition of a perturbation of order one due to the magnetic potential and improving on the usual topology considered in the literature. HOW TO CITE??

Regarding the frequency zero, the bottom of the essential spectrum, I have have been interested in resonances. This time, looking at the problem on $\mathbb{R}^n$, I have studied the property of zero resonant states, that are solutions to $Pu = 0$ which do not belong to L^2. In [] I have considered the operator P as the Laplacian and a multiplicative potential on which we only require integrability assumptions. I have given necessary and sufficient conditions to have an eigenvalue in zero and described the behaviour at infinity of the zero resonant states or eigenstates.

Recently, I have been interested in eigenvalue bounds for the Laplacian with a potential on $\mathbb{R}^n$. In particular, on proving bounds for the number of negative eigenvalues of fractional powers and higher order powers of the Laplacian. This includes the critical case of the Laplacian on $\mathbb{R}^2$. These inequalities are a generalisation on the Cwikel-Lieb-Rosenblum bound which covers the case of the Laplacian with a potential in dimension greater or equal than three. This result is the topic of a work in preparation in collaboration with Sébastien Bréteaux and Jérémy Faupin.

maybe divide as hamiltonians on $\mathbb{R}^n$ and on a manifold?

II. MANY-BODY DYNAMICS

As part of my postdoc, I am currently interested in the study of quantum systems of N particles. In this case the

I. NEUROSCIENCE

The aim of my research on that topic is to understand the links between the different scales describing biological neural networks. I have focused on a McKean-Vlasov model for a noisy and spatially non-homogeneous FitzHugh-Nagumo (FHN) neural network. The model dictates the evolution of the distribution $\mu(t, \boldsymbol{x}, v, w)$ of neurons at time t, position $\boldsymbol{x} \in K$ (K compact set of $\mathbb{R}^d$), with membrane potential $v \in \mathbb{R}$ and adaptation variable $w \in \mathbb{R}$; it reads

$$\partial_t \mu + \partial_v \left(\left(N(v) - w - \mathcal{K}_\Phi[\rho_0 \, \mu] \right) \mu \right) + \partial_w \left(A\,(v, w)\, \mu \right) - \partial_v^2 \mu = 0 \,,$$

with $\rho_0(\boldsymbol{x})$ standing for the spatial distribution of neurons and where the non-linear term $\mathcal{K}_\Phi[\rho_0 \, \mu] \, \mu$ is induced by non-local electrostatic interactions: we suppose that neurons interact through Ohm's law and that the conductance between two neurons is given by an interaction kernel $\Phi : K^2 \to \mathbb{R}$ which depends on their position

$$\mathcal{K}_\Phi[\rho_0 \, \mu](t, \boldsymbol{x}, v) = \int_{K \times \mathbb{R}^2} \Phi(\boldsymbol{x}, \boldsymbol{x}') \, (v - v') \, \rho_0(\boldsymbol{x}') \mu(t, \, \boldsymbol{x}', v', w') \mathrm{d}\boldsymbol{x}' \, \mathrm{d}v' \, \mathrm{d}w' \,.$$

Moreover, the terms N and A are associated to the individual behavior of each neuron, given in our context by the model built by R. FitzHugh and J. Nagumo. I specifically focused on the case where two types of interactions are considered: we decompose the interaction kernel Φ as follows

$$\Phi(\boldsymbol{x}, \boldsymbol{x}') = \Psi(\boldsymbol{x}, \boldsymbol{x}') + \frac{1}{\varepsilon} \, \delta_0(\boldsymbol{x} - \boldsymbol{x}') \,,$$

where the Dirac mass δ_0 accounts for short-range interactions whereas the interaction kernel Ψ models long-range interactions. We analyzed the regime of short-range interactions corresponding to the limit $\varepsilon \ll 1$. We proved that the voltage distribution concentrates into a Dirac mass with Gaussian profile, that is

$$\mu(t, \boldsymbol{x}, v, w) \underset{\varepsilon \to 0}{\sim} \sqrt{\frac{\rho_0(\boldsymbol{x})}{2\,\pi\,\varepsilon}} \exp\left(-\frac{\rho_0(\boldsymbol{x})}{2\,\varepsilon} \left| v - \mathcal{V}(t, \boldsymbol{x}) \right|^2 \right) \otimes \bar{\mu}(t, \boldsymbol{x}, w) \,,$$

where $(\mathcal{V}, \bar{\mu})$ solves the classical FHN reaction-diffusion model

$$\begin{cases} \partial_t \, \mathcal{V} = N(\mathcal{V}) - \mathcal{W} - \mathcal{L}_{\rho_0}[\, \mathcal{V} \,] \,, \\ \partial_t \, \bar{\mu} + \partial_w \left(A(\mathcal{V} \,, \, w) \, \bar{\mu} \right) = 0 \,, \end{cases}$$

where $\mathcal{L}_{\rho_0}\,[\, \mathcal{V} \,]$ is the non local operator induced by Ohmic interactions and with

$$\mathcal{W} = \int_{\mathbb{R}} w \, \bar{\mu}\,(t, \boldsymbol{x}, w) \, \mathrm{d}w \,.$$

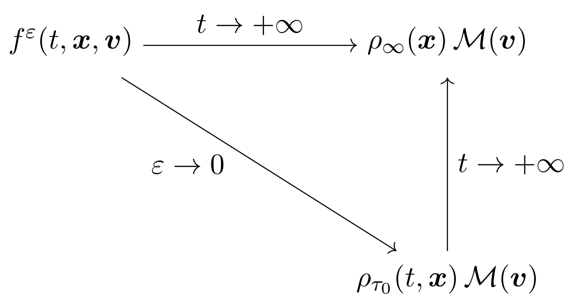
My first contribution on that topic is [4]. It justifies the latter convergence by providing a weak convergence result in probability spaces with explicit and (formally) optimal convergence rates. Then, I completed the latter result by proving quantitative strong convergence estimates in L^1 and L^2 spaces [2]. Furthermore, I also followed a different approach, inspired from the analysis of Hamilton-Jacobi equations, allowing to recover similar results but this time in L^∞-topology [3].

II. KINETIC THEORY

The aim of my research on that topic is to understand the dynamics of a plasma whose charged particles interact through collisions and electrostatic effects. At the mesoscopic level, it may be described by a kinetic equation known as the Vlasov-Poisson-Fokker-Planck model which prescribes the evolution of the density distribution $f^\varepsilon(t, \boldsymbol{x}, \boldsymbol{v})$ depending on time $t \in \mathbb{R}^+$, position $\boldsymbol{x} \in \mathbb{K}^d$ with $\mathbb{K} \in \{\mathbb{T}, \mathbb{R}\}$ and velocity $\boldsymbol{v} \in \mathbb{R}^d$, of electrons in a plasma composed with a single species of ionic positive charges forming a fixed background, whose distribution is given by $\rho_i(\boldsymbol{x})$

$$\begin{cases} \partial_t f^\varepsilon + \dfrac{1}{\varepsilon} \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} f^\varepsilon + \dfrac{1}{\varepsilon} \mathbf{E}^\varepsilon \cdot \nabla_{\boldsymbol{v}} f^\varepsilon = \dfrac{1}{\tau_0 \varepsilon^2} \nabla_{\boldsymbol{v}} \cdot [\, \boldsymbol{v} f^\varepsilon + \nabla_{\boldsymbol{v}} f^\varepsilon \,] , \\[2ex] \mathbf{E}^\varepsilon = -\nabla_{\boldsymbol{x}} \phi^\varepsilon , \quad -\Delta_{\boldsymbol{x}} \phi^\varepsilon = \rho^\varepsilon - \rho_i , \quad \rho^\varepsilon = \displaystyle\int_{\mathbb{R}^d} f^\varepsilon \, \mathrm{d}\boldsymbol{v} , \end{cases} \tag{0.1}$$

where the self-consistent electric field $\mathbf{E}^\varepsilon$ is induced by Coulomb interactions between charges. An overview of the possible dynamics is proposed in the following diagram



where the Maxwellian equilibrium $\mathcal{M}$ is given by

$$\mathcal{M}(v) = (2\pi)^{-d/2} \exp\left(-\frac{|v|^2}{2}\right) ,$$

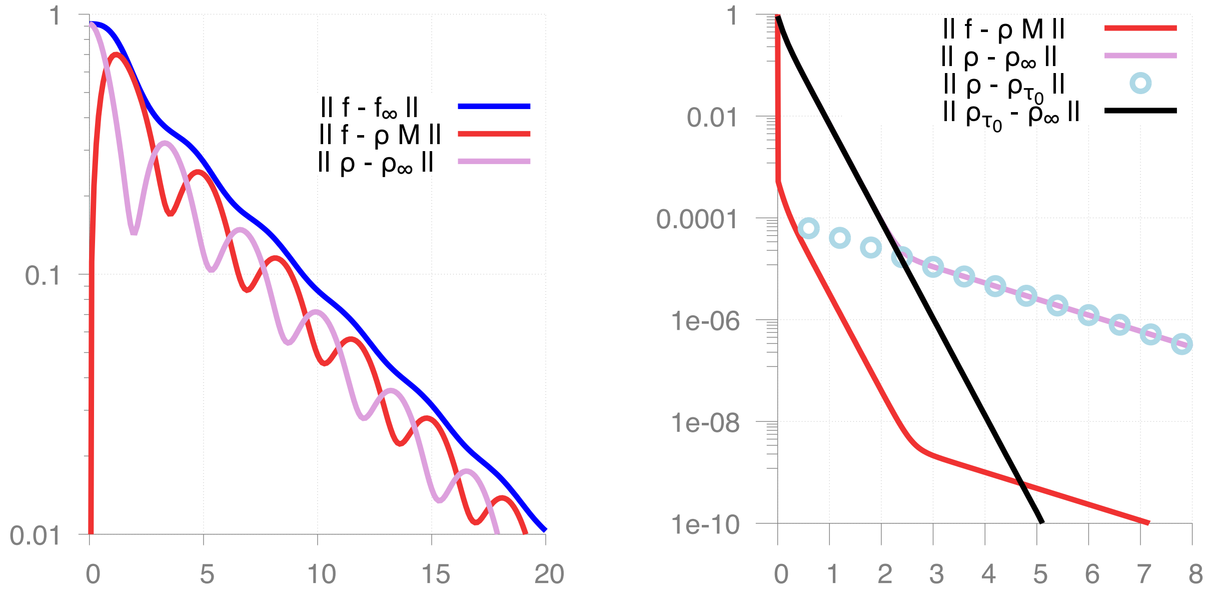
and where ρ solves the following macroscopic model

$$\begin{cases} \partial_t \rho_{\tau_0} + \nabla_{\boldsymbol{x}} \cdot [\, \mathbf{E} \rho_{\tau_0} \,] - \Delta_{\boldsymbol{x}} \rho_{\tau_0} = 0 , \\[2ex] -\Delta_{\boldsymbol{x}} \phi = \rho_{\tau_0} - \rho_i , \quad \mathbf{E} = -\nabla_{\boldsymbol{x}} \phi , \end{cases} \tag{0.2}$$

whereas ρ_∞ is the stationary state of (0.2). My first contribution on that topic is [5]. In this work, we proposed a numerical approximation f of the solution f^ε to (0.1) and proved quantitative estimates ensuring that both the asymptotic $\varepsilon \to 0$ and $t \to +\infty$ are preserved simultaneously by our approximation f in the case where the electric field $\mathbf{E}^\varepsilon$ is replaced by an applied external force field E

Theorem 0.1. *In the linear case where $\mathbf{E}^\varepsilon$ is a given external force field E in (0.1), there exists some $\kappa > 0$ such that for all $n\Delta t \geq 0$ and $\varepsilon > 0$, such that the approximation f and its marginal $\rho = \int f \, \mathrm{d}v$ verify*

$$\|f - \rho_{\tau_0}\mathcal{M}\| \, (n\Delta t) \lesssim \left(\|\rho - \rho_{\tau_0}\| \, (0) + \varepsilon \right) (1 + \kappa\Delta t)^{-n/2} + \left(1 + \varepsilon^{-2} \, \Delta t/2\right)^{-n/2} ,$$



(a) (b)

Figure 1: Time evolution of errors in log scale with $\tau_0 = 5$ and $E(x) = 0.1 * 2\pi\,\cos(2\pi\,x) + 0.9 * 4\pi\,\cos(4\pi\,x)$, when **(a) :** $\varepsilon = 1$ and **(b) :** $\varepsilon = 10^{-4}$.

where ρ_{τ_0} *is the approximation of the macroscopic model* (0.2).

Our efforts to prove such quantitative and uniform result was awarded since it could be illustrated in numerical experiments in which we captured, time boundary layer (Figure 1, (b)), transition phase between hydrodynamic regime and large time behavior (Figure 1, (b)) and also witnessed unexpected phenomena such as oscillatory behaviors (Figure 1, (a)).

Coming back to the full non-linear model (0.1), I proved in [1] a strong convergence result in some weighted L^2-space for the solution to (0.1) as $\varepsilon \to 0$ with explicit and (formally) optimal convergence rate. The result holds in any dimension d, it is non-perturbative and global in time in the limit $\varepsilon \to 0$.

References

[1] A. Blaustein. Diffusive limit of the Vlasov-Poisson-Fokker-Planck model: quantitative and strong convergence results. *SIAM Journal on Mathematical analysis*, submitted.

[2] A. Blaustein. Large coupling in a FitzHug-Nagumo neural network: quantitative and strong convergence results. *Journal of Differential Equations*, submitted.

[3] A. Blaustein and E. Bouin. Concentration profiles in FitzHugh-Nagumo neural networks: A Hopf-Cole approach. *Disc. and Continuous Dyn. Systems Series B*, submitted.

[4] A. Blaustein and F. Filbet. Concentration phenomena in FitzHugh-Nagumo's equations: a mesoscopic approach. *SIAM J. Math. Anal.*, to appear.

[5] A. Blaustein and F. Filbet. On a discrete framework of hypocoercivity for kinetic equations. *Mathematics of Computation*, submitted.